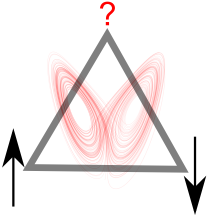
Chaos meets Frustration

Butterfly effect in a classical spin system



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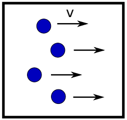
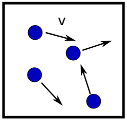
Roderich Moessner (MPI)

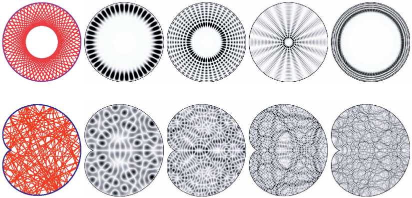
arXiv:1808.02054

Why do we care about chaos?





- **Thermalisation**
- relation to chaos and ergodicity
- regularity seems to oppose thermalisation
- many-body (generic) chaotic



- classical $\leftrightarrow$ QM?

Overview and Connections

Classical Chaos

Butterfly effect
exponential sensitivity
ergodicity
phase-space averages
time averages
QM as $T \rightarrow 0$?

QM-Chaos

EV statistics
ETH
Scars
OTOC's
scrambling
classical limit?

Frustration/Spin-liquids

competing interactions
order suppressed

General Folk-Lore/Expectations

Classical many-body Dynamics

chaotic at $T \to \infty$, order as $T \to 0$
Butterfly-Effect
trajectories diverge

QM many-body dynamics

Unitary/Distance-conserving
but: OTOC's
Quantum bound on chaos (Maldacena '15)

Main Questions

Can we answer these questions in a classical system?

Q: Classical Chaos

- Chaos for $T \in (0, \infty)$?
- low T scaling of chaos?
- OTOC's?

Q: QM Chaos

- Semi-classical limit?
- Quantum-bound?
- OTOC's?

Q: General

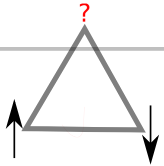
- Relations between microscopic chaos and macroscopic dynamics?

Roadmap

Classical Spin Liquid is ideal platform to explore these issues!

- require chaos at all temperatures
- must suppress order down to $T \to 0$
- need large groundstate manifold

Frustration & Classical Spin-liquids



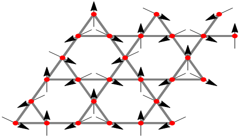
- competing interactions suppress order
- un-ordered, correlated phase

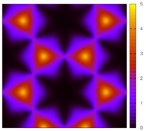
Frustration →

ferromagnetism anti-ferromagnetism ferrimagnetism metamagnetism • • •	geometrical frustration

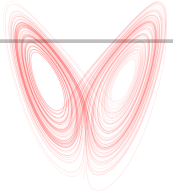
- cooperative paramagnet for $T_{\mathrm{ord}} < T < T_{\mathrm{coop}} \sim J$
- frustration ratio $f = T_{\mathrm{coop}}/T_{\mathrm{ord}} \sim 1 - 100$

- e.g.
  - Kagome (2D)
  - Pyrochlore (3D)
- types
  - U(1)/Coulomb
  - "$\mathbb{Z}_2$" (Rehn '17)
  - "jammed" SL (TB '17)

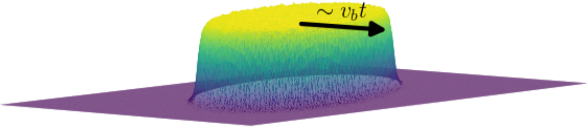




Classical Chaos and Butterfly Effect



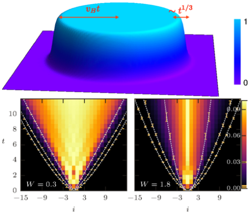
- Butterfly effect
  - exponential dependence on initial conditions (temporal component)
  - (ballistic) propagation (spatial component)
- Lyapunov exponent: growth $\sim e^{\lambda t}$
- butterfly speed: $x \sim v_b t$



OTOC's in QM

OTOC (out-of-time-ordered correlator) (Larkin '69)

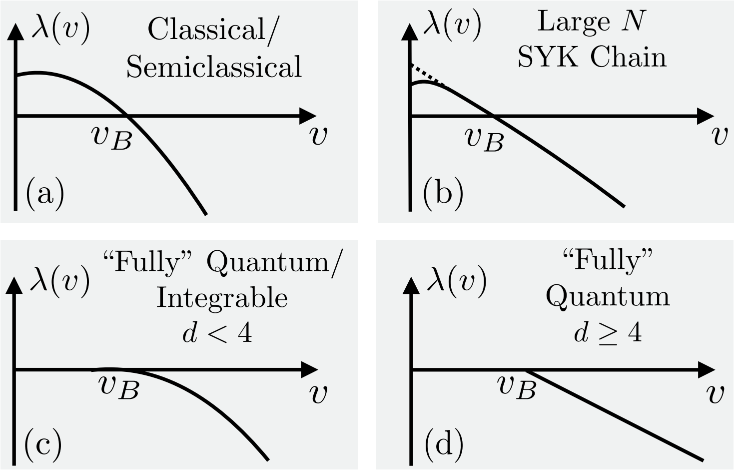
- $\left\langle \left[\hat{W}(t), \hat{V}(0) \right]^2 \right\rangle$
- shows butterfly effect
- Quantum chaos bound $\lambda \lesssim T$ (Maldacena '15)



- random circuits (Nahum '18, Keyserlingk '18)
- random Heisenberg chain (David's talk)
- Luttinger liquids (Dora '17)
- kicked rotor (Galitski '17)
- quantum bound (Maldacena '15)
- black holes (Shenker '14)
- SYK/holography ('93,'15)
- large-N theories
- . . .

Velocity dependent Lyapunov

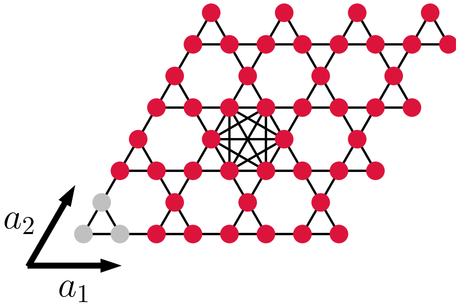
$$D(x,t) \sim e^{\lambda(v=x/t)t}$$



Nahum,arxiv:1803.05902

What we did

Model



- Kagome lattice
- $O(3)$ spins interacts with all spins in a hexagon ⬡

$$H = J \sum_{\mathbf{x},\mathbf{x}' \in \hexagon} \mathbf{S}_\mathbf{x} \cdot \mathbf{S}_{\mathbf{x}'} = \frac{J}{2} \sum_\alpha (\mathbf{L}_\alpha)^2 + \mathrm{const}$$

- Classical "$\mathbb{Z}_2$" spin liquid
  - no order as $T \to 0$

Dynamics

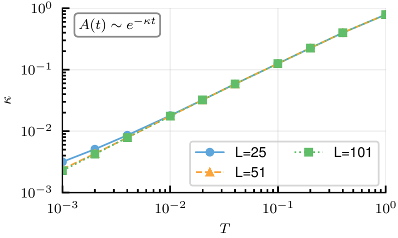
- Precessional Dynamics

$$\frac{d\mathbf{S}_{\mathbf{x}}(t)}{dt} = -\mathbf{S}_{\mathbf{x}}(t) \times \sum_{\mathbf{x}'} J_{\mathbf{x}\mathbf{x}'} \mathbf{S}_{\mathbf{x}'}(t)$$

- Initial states are sampled via Monte-Carlo

Relaxational dynamics

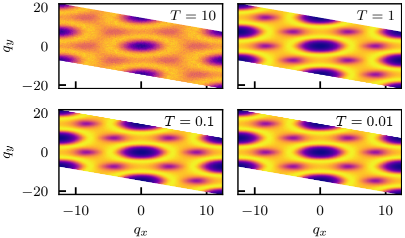
$A(t) \sim e^{-\kappa t}$



No order

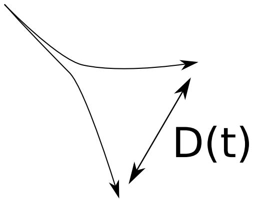
spin diffusion

$\mathcal{S}(\mathbf{q}, \omega) \sim 1/[(Dq^2)^2 + \omega^2]$



Exponential Sensitivity and Chaos

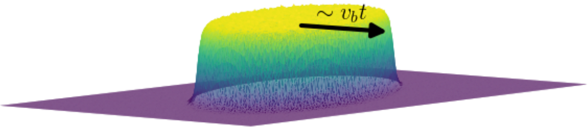
- Perturb initial condition $\delta \mathbf{S}_0(t = 0) = \epsilon(\mathbf{n} \times \mathbf{S}_0)$
- Evolve perturbed state $\tilde{\mathbf{S}}(t) = \mathbf{S}(t) + \delta \mathbf{S}(t)$
- Measure distance $D(t) = \left| \tilde{\mathbf{S}}(t) - \mathbf{S}(t) \right|^2$



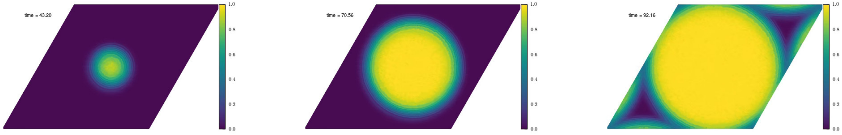
Intuitive Picture/Classical limit of OTOC

$$2D(\mathbf{x}, t) = \left\langle [\delta \mathbf{S}_{\mathbf{x}}(t)]^2 \right\rangle \approx \epsilon^2 \left\langle \{\mathbf{S}_{\mathbf{x}}(t), \mathbf{n} \cdot \mathbf{S}_0(0)\}^2 \right\rangle$$

- intuitive: Distance of perturbed trajectory!
  - $\left|\tilde{\mathbf{S}} - \mathbf{S}\right|^2 = |\delta \mathbf{S}|^2$
- neat: classical version of OTOC!
  - $[\cdots, \cdots] \to \{\cdots, \cdots\}$

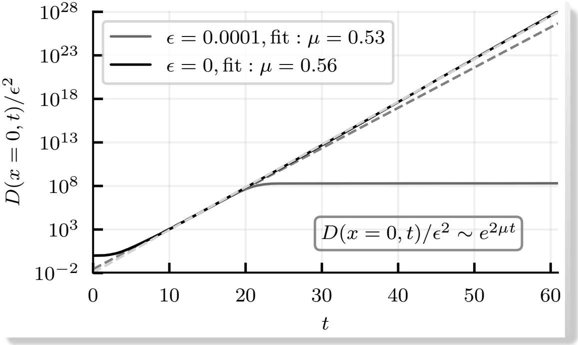


Spreading of OTOC



- Ballistic propagation
- isotropic spreading
- Exponential growth near front

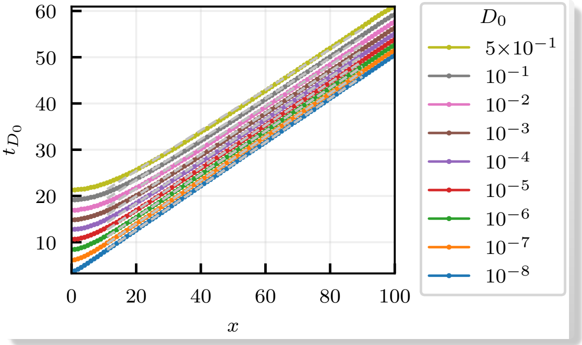
Exponential Growth



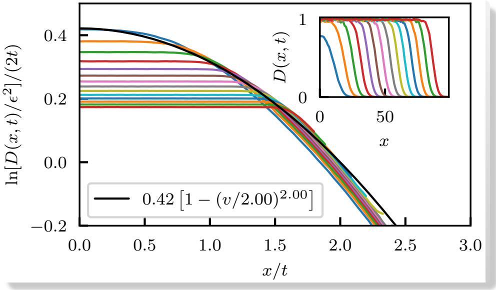
- De-correlator $D(x=0,t) \sim e^{2\mu t}$

Ballistic Propagation

- Arrival times $t(x)$ for which $D(x, t) > D_0$ for threshold D_0
- for ballistic propagation $t(x) = x/v_b$ with butterfly speed v_b



Scaling Form/Velocity dependent Lyapunov

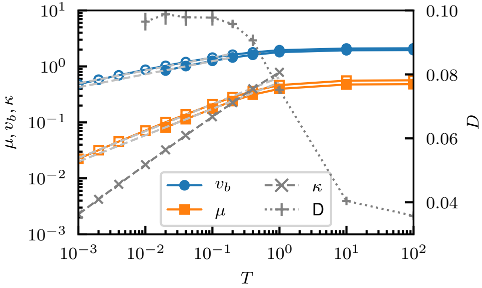


- De-correlator shows scaling collapse

$$D(x,t) \sim \exp[2\mu(1-(v/v_b)^{\nu})t]$$

  along rays $v = x/t$
- Recall: velocity dependent Lyapunov $\lambda(v) = \mu(1 - v/v_b)^{\nu}$

Temperature Dependence of Chaos



at low T

$v_b \sim T^{0.25}$

$\mu \sim T^{0.5}$

$\kappa \sim T$

$D \sim \mathrm{const}$

Butterfly speed v_b *ballistic propagation* of the wavefront

Lyapunov exponent μ *exponential* growth rate of the OTOC

Spin-relaxation rate κ of spin auto-correlation $A(t) \sim e^{-\kappa t}$

Spin diffusion constant D from dynamical structure factor

$\mathcal{S}(\mathbf{q}, \omega) \sim 1/[(Dq^2)^2 + \omega^2]$

Chaos in classical many-body systems

Main results

- Spin-liquid ideal platform to study chaos for $T \in (0, \infty)$ ✓
- Powerlaw scaling of relevant quantities: μ, v_b, κ ✓
  - Connection between macroscopic transport and microscopic chaos $D \sim v_b^2/\mu$ ✓
  - $\mu \sim T^{0.5}$ vanishes slower than quantum bound ✓/?

Directions for future work

- Semi-classical/large-N limit and quantum bound $\mu \sim T$, singular corrections?
- Behaviour of information spreading/entropy?