Artificial Gauge Fields in Cold Atoms: Scattering and Population Dynamics in Floquet Theory

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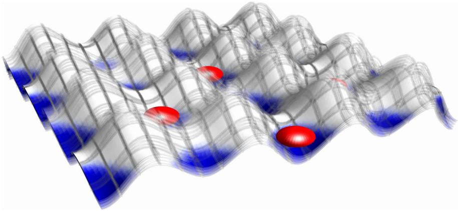
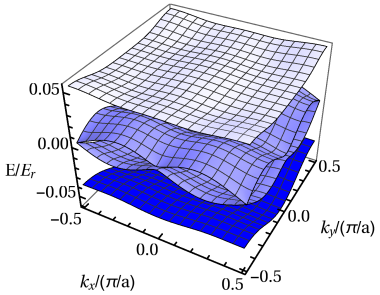


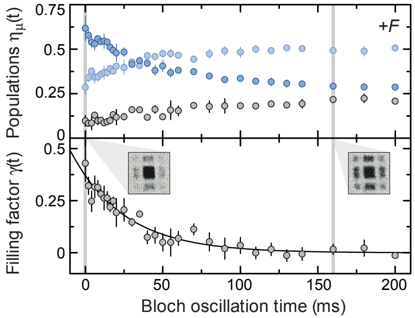
Overview

Artificial gauge fields for cold atoms

- Ultra-cold atomic systems as quantum simulators
- Artificial gauge fields of fundamental interest
  - explore topological physics, QH, FQH
  - exotic non-abelian phases
  - simulation of field theories, QCD, ...
- previous proposals, e.g. rotating the system, limited
- new proposals
  - Time-periodic modulation $\rightarrow$ Floquet
  - Synthetic dimensions

Hofstadter Model and Population Dynamics





Aidelsburger et al, Nature Physics 11, 162-166 (2015)

Why Time-Periodically Driven Quantum Systems

- versatile tool for manipulating quantum systems
- theoretically interesting:
  - non-equilibrium quantum physics
  - retain some properties of time-independent/equilibrium physics
  - long-time behaviour/thermodynamics
  - engineering of band structures
  - transmutation of statistics
  - engineering of many-body-interactions
  - superconductivity
  - ...
- experimentally relevant:
  - artificial gauge fields
  - topological phases
  - probing tool
  - ...

Behaviour of peridically-driven quantum systems

- long-time behavior:
  - integrable systems synchronise and show non-trivial periodic behaviour

    A. Lazarides, A. Das, R. Moessner, PRL 112, 150401
  - non-integrable systems heat up to infinite temperature state

    L. D'Alessio, M. Rigol, PRX 4, 041048;A. Lazarides, A. Das, R. Moessner, PRE 90, 012110
- intermediate times:
  - a pre-thermal state can emerge

    Bukov et al., PRL 115, 205301 (2015); Canovi et al PRE 93, 012130 (2015)
- short times:
  - stability of phases/scattering of Floquet states/resonances

    S. Choudhury and E. J. Mueller, PRA 91, 023624; S. Choudhury and E. J. Mueller, PRA 90, 013621;T.B.,N.R.Cooper,PRA 91, 033601 (2015); PRA 91, 063611 (2015); Bukov et al, Phys. Rev. B 93, 155132 (2016) ;S. Lellocu et al, arXiv:1610.02972 (2016)

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- What are conditions for stability and what are the relevant timescales?

Floquet Theorem

Floquet Theorem

Solutions for Hamiltonians periodic in time $H(t) = H(t+T)$ with period T and associated frequency $\omega = 2\pi/T$ take the form

$$\Psi_\alpha(t) = \exp[-i\epsilon_\alpha t/\hbar]\Phi_\alpha(t)$$

with quasi-energy ϵ_α and the periodic Floquet mode $\Phi_\alpha(t) = \Phi_\alpha(t+T)$.

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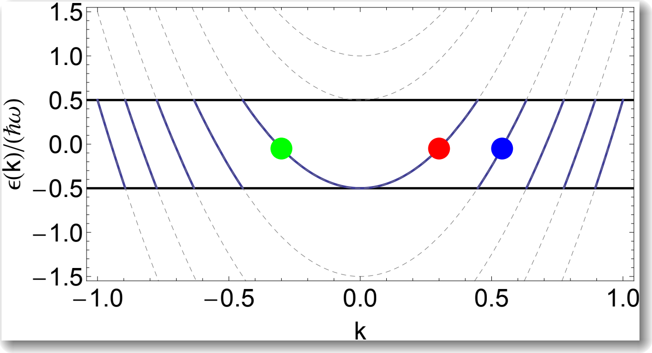
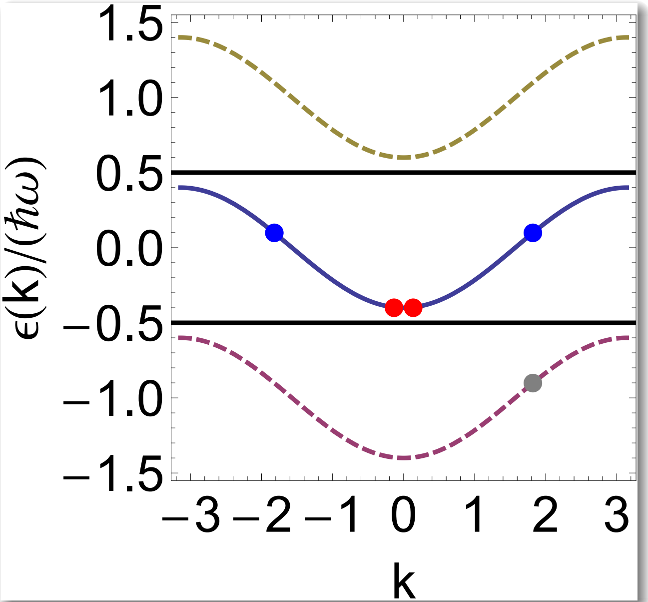
- invariant under $\epsilon_\alpha \to \epsilon_\alpha + n\hbar\omega$, $\Phi_\alpha(t) \to e^{in\omega t}\Phi_\alpha(t)$
  $\to$ Floquet quasi-energy BZ

Floquet Brillouinzone for free unbounded dispersion



Floquet Brillouinzone for bounded (lattice) dispersion



Structure of States

- Floquet modes can be expanded

$$\Psi_\alpha(t) = \exp[-i\epsilon_\alpha t/\hbar] \sum_m \phi_\alpha^m \exp[-im\omega t]$$

- Floquet states behave as if they have multiple energies at $\epsilon_\alpha + m\hbar\omega$ for occupied ϕ_α^m

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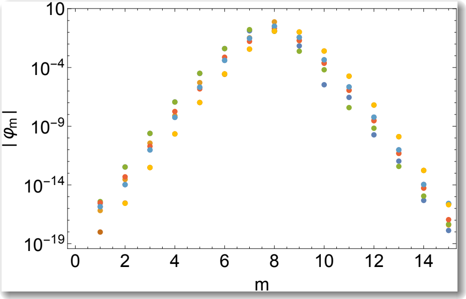
- usually localised in m



General Framework:Floquet Scattering Theory

- Hamiltonian $H(t) = H_0(t) + V$, with time-periodic $H_0(t)$ and static V
- define the interaction picture wrt the time-dependent periodic Hamiltonian $H_0(t)$
- the corresponding propagator is given by

$$U_0(t,t') = \sum_{\alpha} e^{-i\epsilon_\alpha (t'-t)/\hbar} |\Phi_{0,\alpha}(t')\rangle\langle\Phi_{0,\alpha}(t)|$$

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- the interaction becomes time dependent and quasi-periodic

$$V^I(t) = U_0(t,0) V U_0(0,t)$$

- time-dependence depends on the structure of Floquet-states and the interaction

Floquet Fermi Golden Rule (FFGR)

FFGR

$$\gamma_{\mathrm{i}\to\mathrm{f}} = \frac{2\pi}{\hbar}\sum_m |\langle\langle \Phi_{\mathrm{f}}^m \,|\, V \,|\, \Phi_{\mathrm{i}}^0 \rangle\rangle|^2 \delta(\epsilon_{\mathrm{i}}^0 - \epsilon_{\mathrm{f}}^0 - m\hbar\omega)$$

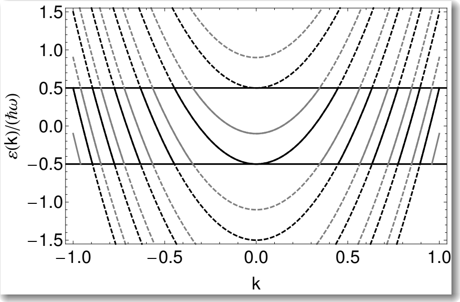
- sum over m corresponds to emission absorption of quanta $\hbar\omega$
- structure of Floquet states and interaction enters via the matrix element $|\langle\langle \Phi_{\mathrm{f}}^m \,|\, V \,|\, \Phi_{\mathrm{i}}^0 \rangle\rangle|^2$

A Simple Single-Particle Toy Model

- Consider a resonantly coupled two-state system in the continuum

$$H_0(t) = \frac{p^2}{2M}\mathbb{1} + \begin{pmatrix} 0 & \Omega e^{-i\omega t} \\ \Omega e^{i\omega t} & -\hbar\omega \end{pmatrix}$$

- with dispersion $\epsilon^m_{k,\tau} = \frac{\hbar^2 k^2}{2M} + \tau\Omega + m\hbar\omega$



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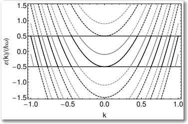


- add scattering off the origin

$$V = \delta(x) \begin{pmatrix} g_0 + g_1 & g_c \\ g_c & g_0 - g_1 \end{pmatrix}$$

- Question: Do we get "inelastic" scattering processes

A Simple Single-Particle Toy Model:Scattering

- the Floquet modes are

$$\Phi_{k,\tau} = \frac{1}{\sqrt{2L}} e^{ikx} \left(\begin{array}{c} 1 \\ \tau e^{i\omega t} \end{array} \right)$$

- the potential in the interaction picture is

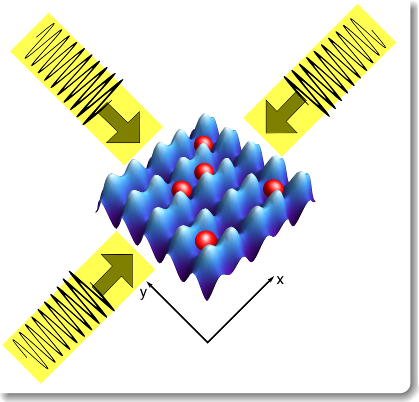
$$V^I(t) = \begin{pmatrix} g_0 & g_1 \\ g_1 & g_0 \end{pmatrix} + g_c \begin{pmatrix} \cos\omega t & i\sin\omega t \\ -i\sin\omega t & \cos\omega t \end{pmatrix} .$$

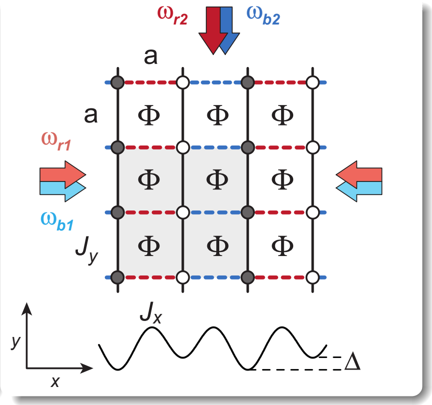
Harper-Hofstadter Model

$$\mathcal{H} = -J \sum_{m,n} \left(e^{in\Phi} a^\dagger_{m+1,n} a_{m,n} + a^\dagger_{m,n+1} a_{m,n} + h.c. \right)$$

- Bosons hopping in a 2D lattice with a complex hopping phase corresponding to a gauge invariant flux Φ
- fractal spectrum $\rightarrow$ Hofstadter-Butterfly
- in low flux limit reduces to continuum Landau levels
- topological bandstructure with non-zero Chern-numbers
- topological transport response, i.e. Quantum Hall effect

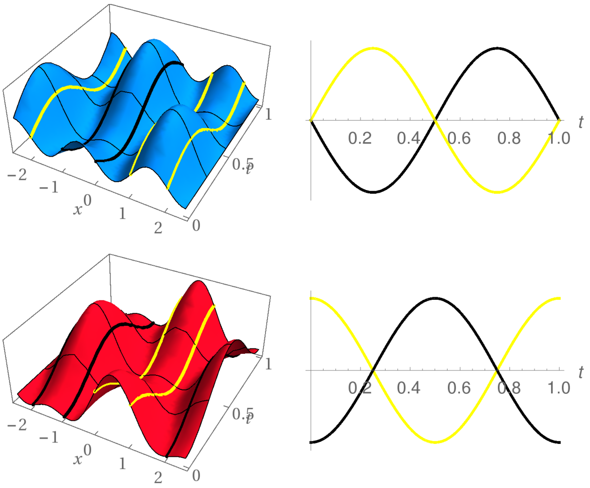
Cold Atom Realisation of Harper-Hofstadter Model





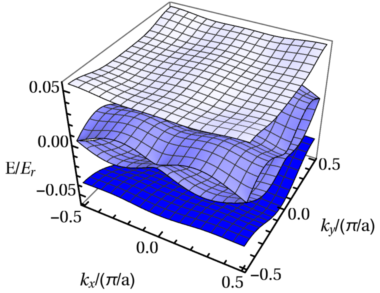
Aidelsburger et al, Nature Physics 11, 162-166 (2015)

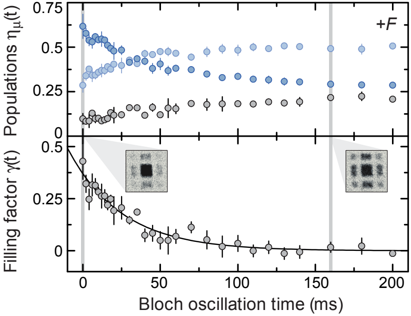
Modulation pattern



- lasers address different bonds
- differential modulation → complex hopping phase

Bandstructure and Population Dynamics





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Optical lattice potential

- The lattice consists of static and oscillating parts

$$V(x,y) = V_{\mathrm{st}}(x,y) + V_{\mathrm{osc}}(x,y,t)$$

- the static part $V(x+2a,y) = V(x,y+a) = V(x,y)$ is periodic with respect to $2a \times 2a$-unit cell
- the oscillating part is quasi-periodic

$$V_{\mathrm{osc}}(r,t) = F(r)e^{i\omega t} + F^*(r)e^{-i\omega t}$$

with quasi-periodic $F(r) = e^{iGr} f(r)$ and $G = (\pi/(2a), \pi/(2a))$

Hofstadter Model: Floquet Matrix

$$H_{\mathrm{F}} = \begin{pmatrix} \ddots & \ddots & & & \\ \ddots & H(k) & F(r) & & \\ & F^*(r) & H(k)+\omega & F(r) & \\ & & F^*(r) & H(k)+2\omega & \ddots \\ & & & \ddots & \ddots \end{pmatrix}$$

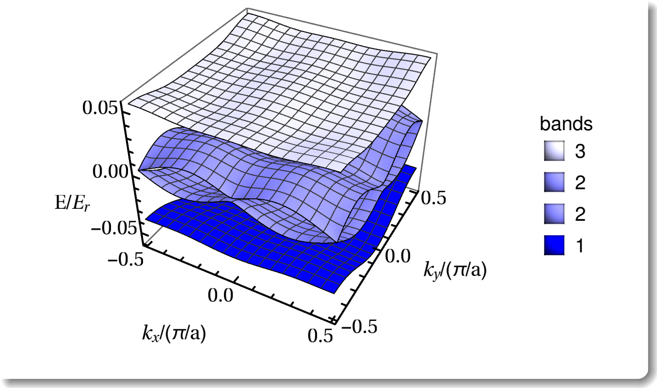
- Expansion of time-dependent Schroedinger equation for Floquet modes in $e^{im\omega t}$, c.f. expansion in plane waves for periodic potential
- $F(r)$ not lattice-periodic

Hofstadter Model: Floquet Matrix gauge-transformed

$$\tilde{\mathrm{H}}_{\mathrm{F}} = \begin{pmatrix} \ddots & \ddots & & & \\ \ddots & H(k) & F(r)e^{iGr} & & \\ & F^*(r)e^{-iGr} & H(k+G)+\omega & F(r)e^{-iGr} & \\ & & F^*(r)e^{iGr} & H(k)+2\omega & \ddots \\ & & & \ddots & \ddots \end{pmatrix}$$

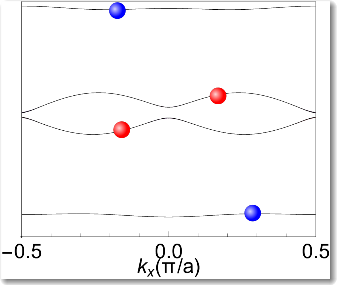
- $e^{\pm iGr}F(r)$ is lattice-periodic with respect to $(2a) \times (2a)$ unit-cell
- can expand in Bloch-bands of static Hamiltonian $\rightarrow$ 4 bands at k and 4 bands at $k+G$
- Symmetry under $\tilde{\mathrm{H}}_{\mathrm{F}} \rightarrow \tilde{\mathrm{H}}_{\mathrm{F}} + \omega$, $k \rightarrow k+G$ reduces to the 4 Hofstadter bands
- each eigenstate is a time-periodic superposition of the 8 bands of the static Hamiltonian

Hofstadter Model: Bandstructure



Hofstadter Model: Scattering Processes

- inelastic process
  requiring no energy absorption $(2, 2) \rightarrow (1, 3)$ and reverse $(1, 3) \rightarrow (2, 2)$



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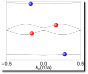
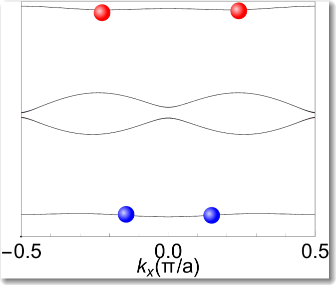
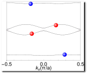


- inelastic process requiring no energy absorption $({\color{red}3},{\color{red}3}) \rightarrow ({\color{blue}1},{\color{blue}1})$

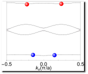


Hofstadter Model: Scattering Processes

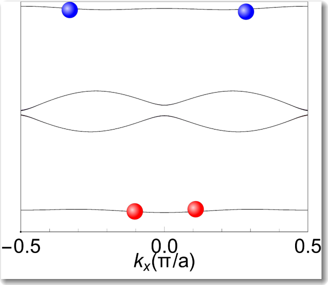
- inelastic process requiring no energy absorption $(2,2) \to (1,3)$ and reverse $(1,3) \to (2,2)$



- inelastic process requiring no energy absorption $(3,3) \to (1,1)$



- inelastic process requiring energy absorption $(1,1) \to (3,3)$; only allowed within Floquet picture

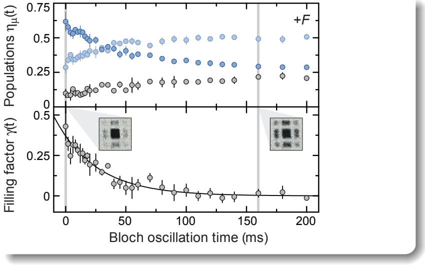


Rate Model

- model full experimental system, no free parameter
- incoherent homogeneous initial population spread over total BZ
- in particular, not condensed!
- compute transition rates from the FFGR, average over initial momenta

$$\frac{dn_\alpha}{d\tau} = \sum_{\alpha',\beta',\gamma',\delta'} \gamma^{\mathrm{av}}_{(\alpha',\beta')\to(\gamma',\delta')} \left[(\delta_{\alpha,\gamma'} + \delta_{\alpha,\delta'}) - (\delta_{\alpha,\alpha'} + \delta_{\alpha,\beta'})\right] n_{\alpha'} n_{\beta'}$$

Hofstadter Model: Population Dynamics (without Floquet)



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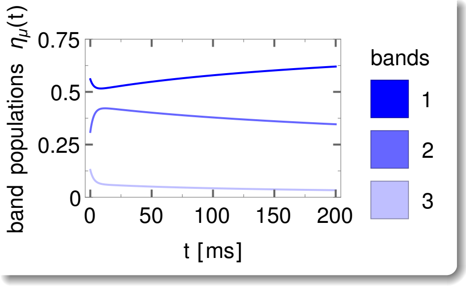
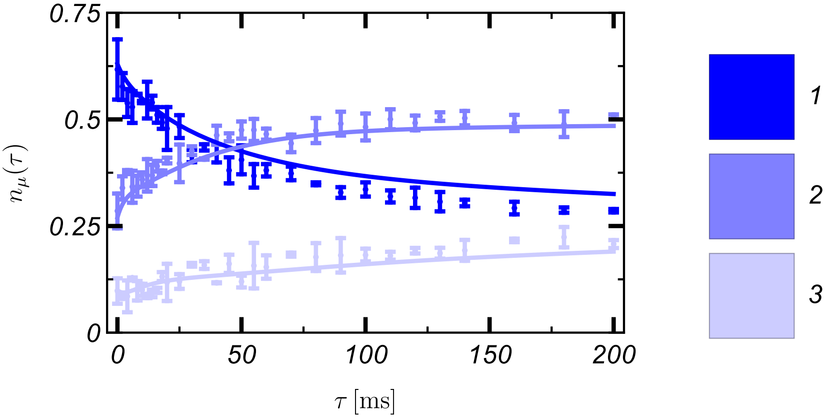


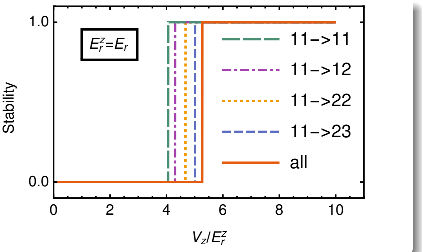
FFGR

Hofstadter Model: Floquet Population Dynamics



T.B., N.R.Cooper, PRA 91, 063611 (2015)

Suppression of scattering by transverse confinement



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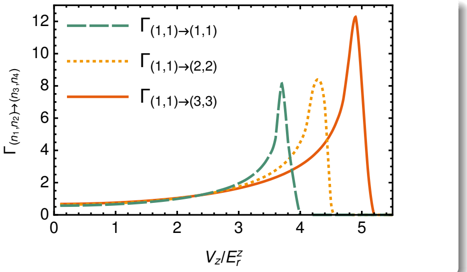
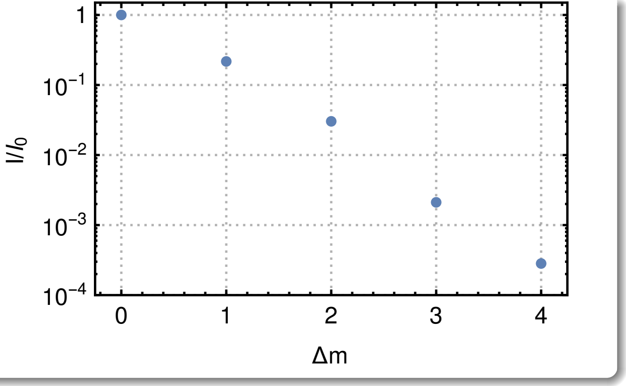


T.B.,N.R.Cooper, PRA 91, 063611 (2015)

- suppression of heating $\rightarrow$ access interacting regime and fractional QH

Hofstadter Model: Higher Order Scattering rates



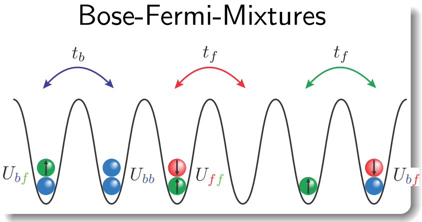
Conclusions

Floquet systems show genuinely different behaviour

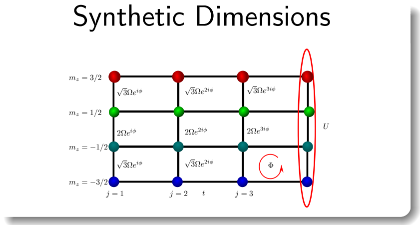
- effective bandstructure + interactions requires careful interpretation
- Floquet heating and interband-transitions
- can be suppressed by transverse confinement
- $\rightarrow$ (strongly) interacting cold gas systems in artificial gauge fields

Thank you for your attention

I've also worked on



exotic superconductivity within DCA using CT-QMC



(pair) superfluid phases in synthetic dimensions with SU(N) interactions using DMRG