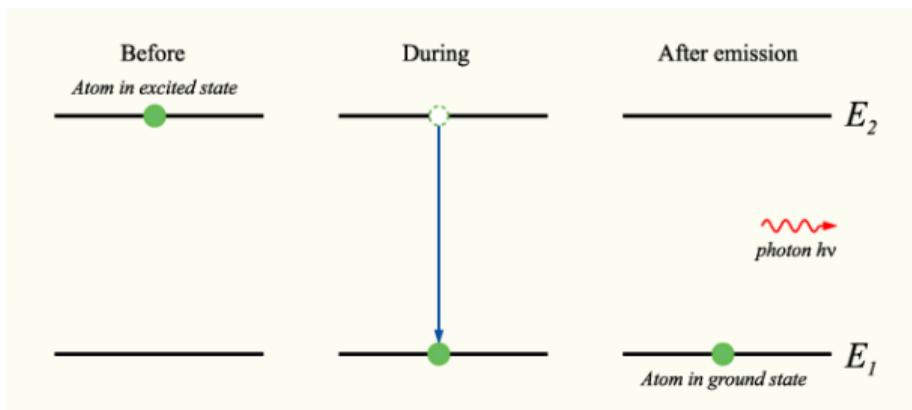


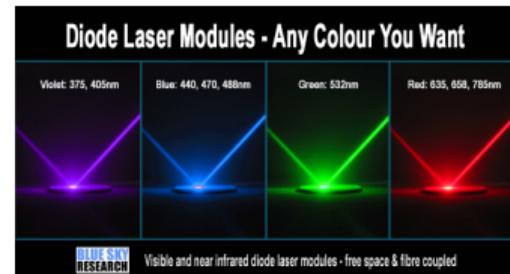
**Understanding the interplay between Pauli blocking,
dipolar interactions and atomic motion in a long-lived 2D
Fermi gas**

(Spontaneous) radiative decay

Radiative decay one of the most fundamental and ubiquitous processes in nature



- ▶ Fireflies
- ▶ Laser diodes
- ▶ Gamma decay of nuclei
- ▶ ...



Systems of matter and light

Atom-Light Hamiltonian

$$H_M + H_{LM} + H_L$$

H_M

- ▶ Matter fields
- ▶ Fermions c^\dagger
- ▶ Bosons b^\dagger
- ▶ level structure

H_{LM}

- ▶ Coupling between Light and Matter

H_L

- ▶ (bosonic) photon modes a^\dagger

Systems of matter and light

Atom-Light Hamiltonian

$$H_M + H_{LM} + H_L$$

H_M

- ▶ Matter fields
- ▶ Fermions c^\dagger
- ▶ Bosons b^\dagger
- ▶ level structure

H_{LM}

- ▶ Coupling between Light and Matter

H_L

- ▶ (bosonic) photon modes a^\dagger

Engineering of AMO systems

To a large degree all we are doing is manipulating different parts of this

Atom-Light interaction

$$H_{LM}$$

$$e^{i\mathbf{q}\hat{\mathbf{R}}}\hat{\sigma}^+(\hat{\mathbf{R}})\hat{a}_{\mathbf{q}} + c.c.$$

- ▶ couples internal state and motional state of atom to photon field
- ▶ describes emission/absorption of photon with momentum \mathbf{q} from/into the field modes

Atom-Light interaction

$$H_{LM}$$

$$e^{i\mathbf{q}\hat{\mathbf{R}}}\hat{\sigma}^+(\hat{\mathbf{R}})\hat{a}_{\mathbf{q}} + c.c.$$

- ▶ couples internal state and motional state of atom to photon field
- ▶ describes emission/absorption of photon with momentum \mathbf{q} from/into the field modes

Momentum-Kick

- ▶ Laser-Cooling
- ▶ Momentum resolved scattering

Internal state

- ▶ optical control
- ▶ optical potentials
- ▶ Stark shifts
- ▶ atomic clocks

Light part

- ▶ mode-spectrum
- ▶ wave-guides

Engineering of atom-light coupling

$$H = H_M + H_L + H_{LM}$$
$$H_{LM} \sim e^{iq\hat{R}} \hat{\sigma}^+(\hat{R}) \hat{a}_{\mathbf{q}} + c.c.$$

Light part

- ▶ change field spectrum
- ▶ Purcell effect
- ▶ **Cavity QED, waveguide QED**

Level Structure

- ▶ multi-level systems
- ▶ dressing of internal states
- ▶ allowed/forbidden transitions

atomic motional spectrum

- ▶ optical potentials
- ▶ confinement

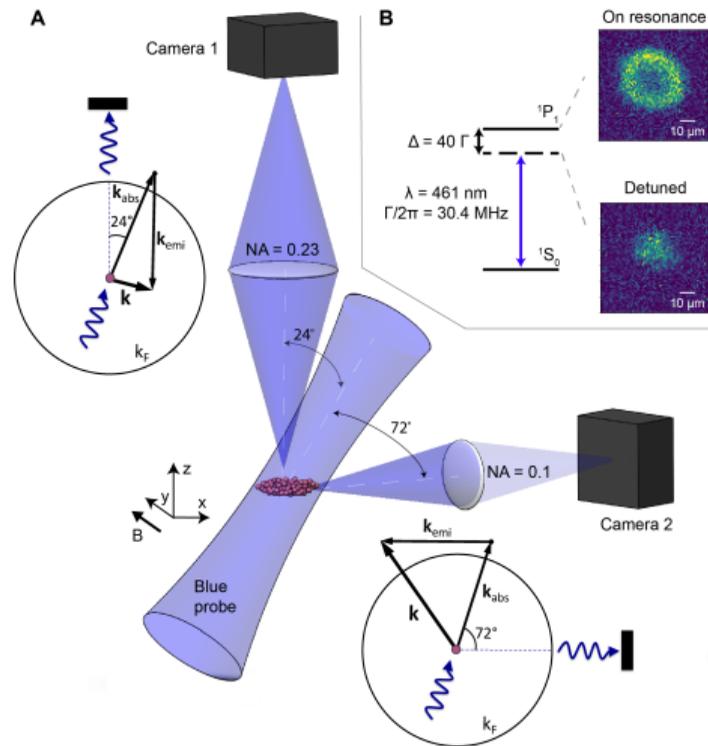
Available states

- ▶ Quantum-Statistics
- ▶ **Pauli blocking**
- ▶ Bose enhancement

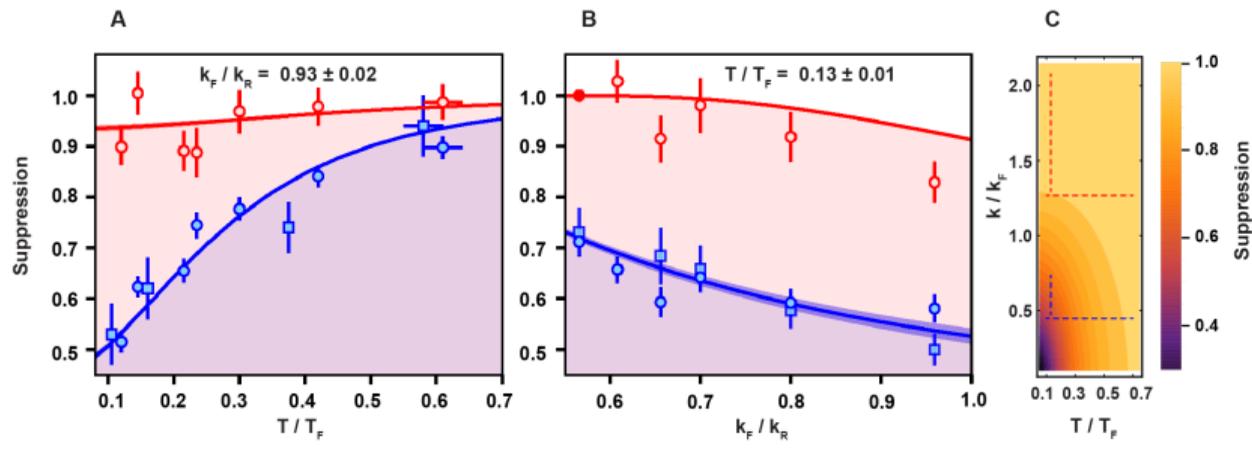
Recent Experimental Results

- ▶ Observation of reduced light-scattering from atomic ensembles
 - ▶ Ye et al, "Pauli blocking of atomic spontaneous decay" (**3.3.2021**)
 - ▶ Ketterle et al, "Pauli blocking of light scattering in degenerate fermions" (**11.3.2021**)
 - ▶ Kjærgaard et al, "Observation of Pauli blocking in light scattering from quantum degenerate fermions" (**3.3.2021**)

Experimental Work II



Experimental Work II



¹ Ye et al (2021)

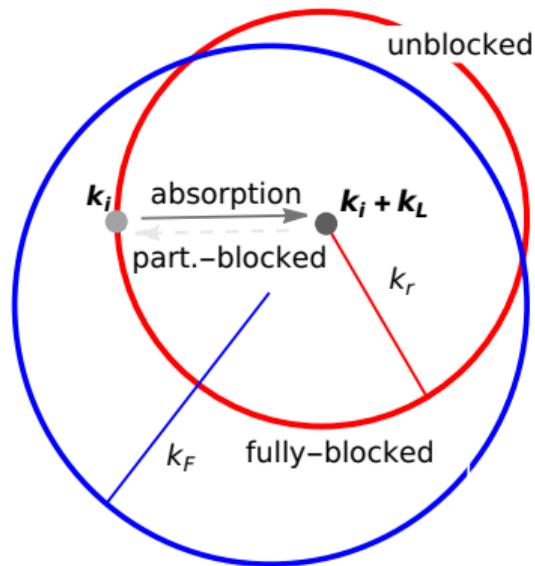
Missing Piece

Observation of Pauli blocking/enhanced lifetimes in population measurements

Challenge

- ▶ Transition/Decay has to be slow to be measured!
- ▶ then: motional, interaction and dipolar time-scales are comparable
- ▶ → need to treat all properly

Basic idea of Pauli-Blocking of spontaneous decay



Scenario

- ▶ excited atom in momentum \mathbf{k} embedded in a Fermi-Sea
- ▶ decays onto states with $\mathbf{q} = \mathbf{k} + \mathbf{k}_r$
- ▶ blocked if state \mathbf{q} occupied

Consequences

- ▶ Emission is directional

Relevant Scales

- ▶ Size of Fermi - Sea: k_F, N
- ▶ momentum kick/recoil: ratio k_r/k_F

Non-interacting description

Basic assumptions

- ▶ non-interacting particles
- ▶ single excitation
- ▶ Fermi sea

Initial State

- ▶ FS with one hole at \mathbf{k}_i , and one excitation at $\mathbf{k}_i + \mathbf{k}_L$
- ▶ will ignore presence of hole

Scales

- ▶ k_0 recoil momentum
- ▶ k_F Fermi-momentum
- ▶ $\mathbf{k}_L = k_0 \mathbf{e}_x$: excitation "kick" by laser

Final State

- ▶ excitation decays to a momentum on a sphere with radius k_0 (weighted by emission pattern of transition)

$$S(\mathbf{k}) = \frac{h^{-3} \int d^3\mathbf{p} d^3\mathbf{q} n_{FD,i}(\mathbf{p}, \mathbf{q}) [1 - n_{FD,f}(\mathbf{p} + \hbar(\mathbf{k} + \mathbf{k}_L), \mathbf{q})]}{h^{-3} \int d^3\mathbf{p} d^3\mathbf{q} n_i(\mathbf{p}, \mathbf{q})}$$

$$M = \int d^2k P(\mathbf{k}) S(\mathbf{k})$$

$$n_{FD}(\mathbf{p}, \mathbf{q}) = \frac{1}{1 + 1/z e^{\beta[\sum_i m\omega_i^2 q_i^2/2 + \sum_i p_i^2/(2m)]}}$$

- ▶ uses semi-classical phase space formulation
- ▶ have to add in dipole-emission pattern $P(\mathbf{k})$ "by-hand"
- ▶ **cannot include interactions**

¹ Thywissen et al. (2009), Zoller et al (1998), Busch et al (2009)

Fundamental Challenge of Interactions

In the regime of strong Pauli blocking interactions are not negligible

Pauli-Blocking

Need: $k_F/k_0 \gtrsim 1$

OD Scaling (3D)

Want: $OD = \int dz n \sigma \ll 1$

$$\sigma \sim \lambda^2 \sim 1/k_0^2$$

$$n \sim k_F^3$$

$$OD \sim k_F(k_F/k_0)^2 \gg 1$$

Current State of affairs

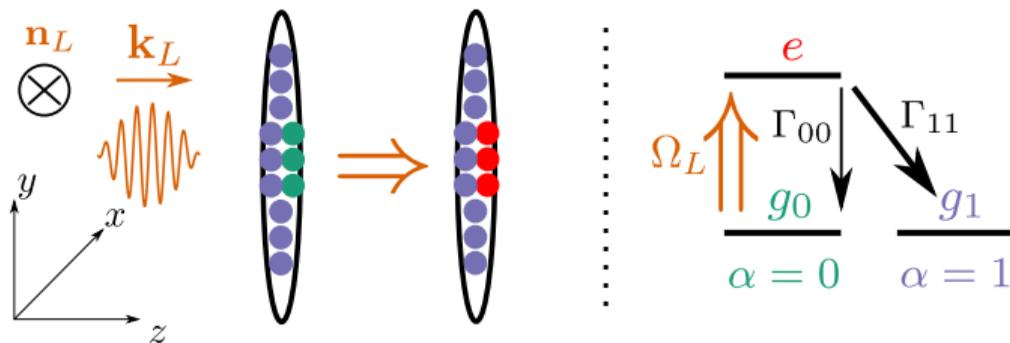
- ▶ can describe **non-interacting** Pauli-Blocking
- ▶ Strong Pauli blocking \leftrightarrow interacting regime

- ▶ can describe interacting atoms (**without motion**) \rightarrow frozen-atom coupled dipole
- ▶ Correct description of Pauli blocking \leftrightarrow motion (momentum-kick)

Ambitious Goal

Understand the interplay of **Pauli blocking, dipolar interactions and motion** in quantum degenerate Fermi gases

Our System/Jun's work II



Key-Properties

- ▶ Strong 2D confinement/Lamb-Dicke
 - ▶ no initial momentum kick
 - ▶ smaller state space
- ▶ Λ level structure
 - ▶ only 3 internal levels
 - ▶ useful Γ_{00}/Γ_{11} -ratio
 - ▶ can decouple excited and blocking species

Our starting point

$$H_M + H_{LM} + H_L$$

Main approximation

- ▶ Integrate out light-field
- ▶ Born-Markov

Multi-level dipolar master equation

$$\dot{\hat{\rho}} = -i [\hat{H}, \hat{\rho}] + \mathcal{L}(\hat{\rho})$$

1

ME in real space

$$\hat{H} = - \sum_{\alpha, \beta} \int d\mathbf{r} d\mathbf{r}' \left(\mathbf{d}_\alpha \cdot \text{Re } G(\mathbf{r} - \mathbf{r}') \cdot \bar{\mathbf{d}}_\beta \right) \hat{\sigma}_{e\mathbf{g}_\alpha}(\mathbf{r}) \hat{\sigma}_{\mathbf{g}_\beta e}(\mathbf{r}') \quad (1)$$

$$\mathcal{L}(\hat{\rho}) = - \sum_{\alpha, \beta} \int d\mathbf{r} d\mathbf{r}' \left(\mathbf{d}_\alpha \cdot \text{Im } G(\mathbf{r} - \mathbf{r}') \cdot \bar{\mathbf{d}}_\beta \right) \left(\left\{ \hat{\sigma}_{e\mathbf{g}_\alpha}(\mathbf{r}) \hat{\sigma}_{\mathbf{g}_\beta e}(\mathbf{r}'), \hat{\rho} \right\} - 2 \hat{\sigma}_{\mathbf{g}_\beta e}(\mathbf{r}') \hat{\rho} \hat{\sigma}_{e\mathbf{g}_\alpha}(\mathbf{r}) \right) \quad (2)$$

$$G(\mathbf{r}) = \frac{3\Gamma}{4} \left\{ [\mathbf{I} - \hat{\mathbf{r}} \otimes \hat{\mathbf{r}}] \frac{e^{ik_0 r}}{k_0 r} [\mathbf{I} - 3\hat{\mathbf{r}} \otimes \hat{\mathbf{r}}] \left[\frac{ie^{ik_0 r}}{(k_0 r)^2} - \frac{e^{ik_0 r}}{(k_0 r)^3} \right] \right\}, \quad (3)$$

Deriving ME in momentum space

- ▶ Expand ME in single-particle basis $\psi^\dagger(\mathbf{r}) = \sum_{\mathbf{k}} \bar{\phi}_{\mathbf{k}} c_{\mathbf{k}}^\dagger$

Single particle Basis

$$\phi_{\mathbf{k}}(x, y, z) = \frac{1}{\sqrt{A}} e^{i(k_x x + k_y y)} \psi_0(z)$$

- ▶ only keep "resonant" terms $(ijkl) \rightarrow (ij, ij)$ or $(ijkl) \rightarrow (ij, ji)$
- ▶ "spin-model like"

Expansion in k-basis

$$\hat{H} = \sum_{\alpha,\beta} \sum_{ijkl} \Delta_{\alpha\beta}^{ij,kl} \hat{c}_{e,i}^\dagger \hat{c}_{g_\beta,k}^\dagger \hat{c}_{g_\alpha,j} \hat{c}_{e,l}, \quad (4)$$

$$\mathcal{L}(\hat{\rho}) = - \sum_{\alpha,\beta} \sum_{ijkl} \Gamma_{\alpha\beta}^{ij,kl} \left(\left\{ \hat{\sigma}_{eg_\alpha}^{ij} \hat{\sigma}_{g_\beta e}^{kl}, \hat{\rho} \right\} - 2 \hat{\sigma}_{g_\beta e}^{kl} \hat{\rho} \hat{\sigma}_{eg_\alpha}^{ij} \right), \quad (5)$$

$$\Delta_{\alpha\beta}^{ij,kl} = \int d\mathbf{r} d\mathbf{r}' \bar{\phi}_i(\mathbf{r}) \phi_j(\mathbf{r}) \operatorname{Re} G_{\alpha\beta}(\mathbf{r} - \mathbf{r}') \bar{\phi}_k(\mathbf{r}') \phi_l(\mathbf{r}'), \quad (6)$$

$$\Gamma_{\alpha\beta}^{ij,kl} = \int d\mathbf{r} d\mathbf{r}' \bar{\phi}_i(\mathbf{r}) \phi_j(\mathbf{r}) \operatorname{Im} G_{\alpha\beta}(\mathbf{r} - \mathbf{r}') \bar{\phi}_k(\mathbf{r}') \phi_l(\mathbf{r}'), \quad (7)$$

$$G(\mathbf{r}) = \frac{3\Gamma}{4} \left\{ [\mathbf{I} - \hat{\mathbf{r}} \otimes \hat{\mathbf{r}}] \frac{e^{ik_0 r}}{k_0 r} [\mathbf{I} - 3\hat{\mathbf{r}} \otimes \hat{\mathbf{r}}] \left[\frac{ie^{ik_0 r}}{(k_0 r)^2} - \frac{e^{ik_0 r}}{(k_0 r)^3} \right] \right\}, \quad (8)$$

Master equation in momentum space

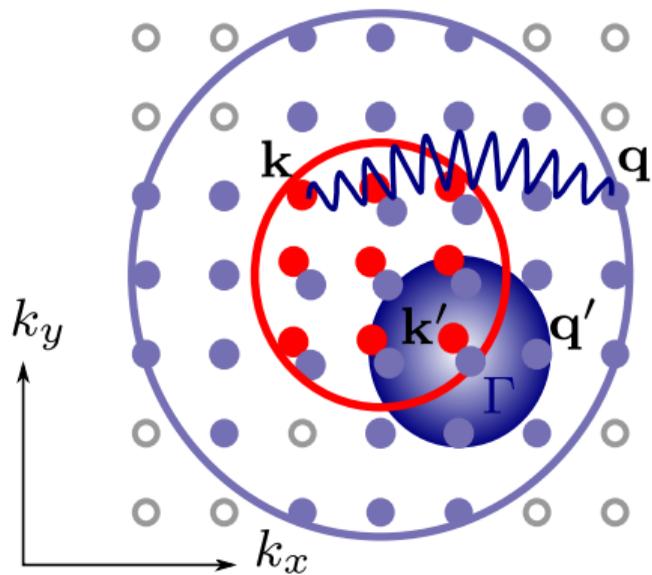
Multi-level dipolar master equation

$$\dot{\hat{\rho}} = -i [\hat{H}, \hat{\rho}] + \mathcal{L}(\hat{\rho})$$

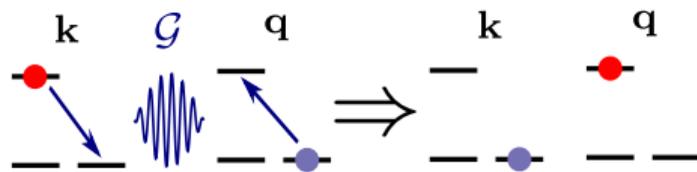
$$\begin{aligned} \hat{H} = \sum_{\alpha, \beta} \left(\sum_{\mathbf{k}, \mathbf{q}} \Delta_{\alpha\beta}^{\mathbf{k}\mathbf{k}, \mathbf{q}\mathbf{q}} \hat{c}_{e, \mathbf{k}}^\dagger \hat{c}_{g\beta, \mathbf{q}}^\dagger \hat{c}_{g\alpha, \mathbf{k}} \hat{c}_{e, \mathbf{q}} \right. \\ \left. + \sum_{\mathbf{k} \neq \mathbf{q}} \Delta_{\alpha\beta}^{\mathbf{k}\mathbf{q}, \mathbf{q}\mathbf{k}} \hat{c}_{e, \mathbf{k}}^\dagger \hat{c}_{g\beta, \mathbf{q}}^\dagger \hat{c}_{g\alpha, \mathbf{q}} \hat{c}_{e, \mathbf{k}} \right) \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{L}(\hat{\rho}) = - \sum_{\alpha, \beta} \left(\sum_{\mathbf{k}, \mathbf{q}} \Gamma_{\alpha\beta}^{\mathbf{k}\mathbf{k}, \mathbf{q}\mathbf{q}} \left(\left\{ \hat{\sigma}_{eg\alpha}^{\mathbf{k}\mathbf{k}} \hat{\sigma}_{g\beta e}^{\mathbf{q}\mathbf{q}}, \hat{\rho} \right\} - 2 \hat{\sigma}_{g\beta e}^{\mathbf{q}\mathbf{q}} \hat{\rho} \hat{\sigma}_{eg\alpha}^{\mathbf{k}\mathbf{k}} \right) \right. \\ \left. + \sum_{\mathbf{k} \neq \mathbf{q}} \Gamma_{\alpha\beta}^{\mathbf{k}\mathbf{q}, \mathbf{q}\mathbf{k}} \left(\left\{ \hat{\sigma}_{eg\alpha}^{\mathbf{k}\mathbf{q}} \hat{\sigma}_{g\beta e}^{\mathbf{q}\mathbf{k}}, \hat{\rho} \right\} - 2 \hat{\sigma}_{g\beta e}^{\mathbf{q}\mathbf{k}} \hat{\rho} \hat{\sigma}_{eg\alpha}^{\mathbf{k}\mathbf{q}} \right) \right) \end{aligned} \quad (10)$$

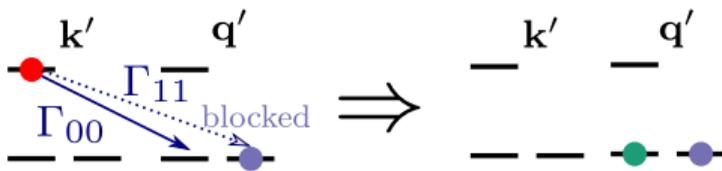
ME in momentum space



Dipolar Exchange



Spontaneous Decay



Technical Detail

Ultimately, we'd like to describe a trapped gas

2D trapped

- ▶ States $|n_x, n_y\rangle$
- ▶ $E = \hbar\omega_{\perp}(n_x + n_y + 1)$
- ▶ $E_F = \hbar\omega_{\perp}\sqrt{2N}$
- ▶ N, ω_{\perp}

2D Box

- ▶ States $|k_x, k_y\rangle$
- ▶ $E = \hbar(k_x^2 + k_y^2)/(2M)$
- ▶ $E_F = \hbar k_F^2/(2M)$
- ▶ $N, \text{Area of box } A$

Consequences

- ▶ Different density of states
- ▶ Need to choose A properly to approximate trapped gas

Equations of motion

- ▶ Focus on momentum diagonal elements $\rho_{\mathbf{q}\mathbf{q}}^{\mu\nu} = \langle \hat{c}_{\mu,\mathbf{q}}^\dagger \hat{c}_{\nu,\mathbf{q}} \rangle$

Key approximation

Factorise 4 operator products

$$\langle \hat{c}_{\mu,i}^\dagger \hat{c}_{\nu,j}^\dagger \hat{c}_{\mu',k} \hat{c}_{\nu',l} \rangle \approx \delta_{il} \delta_{jk} \langle \hat{\sigma}_{il}^{\mu\nu'} \rangle \langle \hat{\sigma}_{jk}^{\nu\mu'} \rangle - \delta_{ik} \delta_{jl} \langle \hat{\sigma}_{ik}^{\mu\mu'} \rangle \langle \hat{\sigma}_{jl}^{\nu\nu'} \rangle,$$

Justification

- ▶ Initial state has no momentum off-diagonal correlations
- ▶ valid at short times

Evolution of the excited state

$$\frac{d\rho_{\mathbf{q}\mathbf{q}}^{ee}}{dt} = \sum_{\alpha} \sum_k -2(1 - \rho_{\mathbf{k}\mathbf{k}}^{g_{\alpha}g_{\alpha}}) \rho_{\mathbf{q}\mathbf{q}}^{ee} \Gamma_{\alpha\alpha}^{\mathbf{k}\mathbf{q},\mathbf{q}\mathbf{k}} \quad (11)$$

$$+ \sum_{\alpha,\beta} \sum_k i \left(\rho_{\mathbf{q}\mathbf{q}}^{eg_{\alpha}} \rho_{\mathbf{k}\mathbf{k}}^{g_{\beta}e} \mathcal{G}_{\alpha\beta}^{\mathbf{q}\mathbf{q},\mathbf{k}\mathbf{k}} - \rho_{\mathbf{q}\mathbf{q}}^{g_{\beta}e} \rho_{\mathbf{k}\mathbf{k}}^{eg_{\alpha}} \bar{\mathcal{G}}_{\alpha\beta}^{\mathbf{q}\mathbf{q},\mathbf{k}\mathbf{k}} \right) \quad (12)$$

Key Observation

- ▶ First Line → Pauli blocked decay
 - ▶ $|e, \mathbf{q}, n_{0,z}\rangle \rightarrow |g_{\alpha}, \mathbf{k}, n_{0,z}\rangle$ mediated by $\Gamma_{\alpha\alpha}^{\mathbf{k}\mathbf{q},\mathbf{q}\mathbf{k}}$
 - ▶ Pauli suppressed by $1 - \rho_{\mathbf{k}\mathbf{k}}^{g_{\alpha}g_{\alpha}}$
- ▶ Second Line → Super/Sub-radiance and cooperative effects
 - ▶ depends on coherences $\rho^{eg_{\alpha}}$
 - ▶ has coherent and incoherent part: $\mathcal{G}_{\alpha\beta}^{\mathbf{k}\mathbf{k},\mathbf{q}\mathbf{q}} = \Delta_{\alpha\beta}^{\mathbf{k}\mathbf{k},\mathbf{q}\mathbf{q}} + i\Gamma_{\alpha\beta}^{\mathbf{k}\mathbf{k},\mathbf{q}\mathbf{q}}$

Non-interacting Part

ME

$$\frac{d\rho_{\mathbf{q}\mathbf{q}}^{ee}}{dt} = \sum_{\alpha} \sum_k -2(1 - \rho_{\mathbf{k}\mathbf{k}}^{g_{\alpha}g_{\alpha}}) \rho_{\mathbf{q}\mathbf{q}}^{ee} \Gamma_{\alpha\alpha}^{\mathbf{k}\mathbf{q},\mathbf{q}\mathbf{k}}$$

Semi-Classics

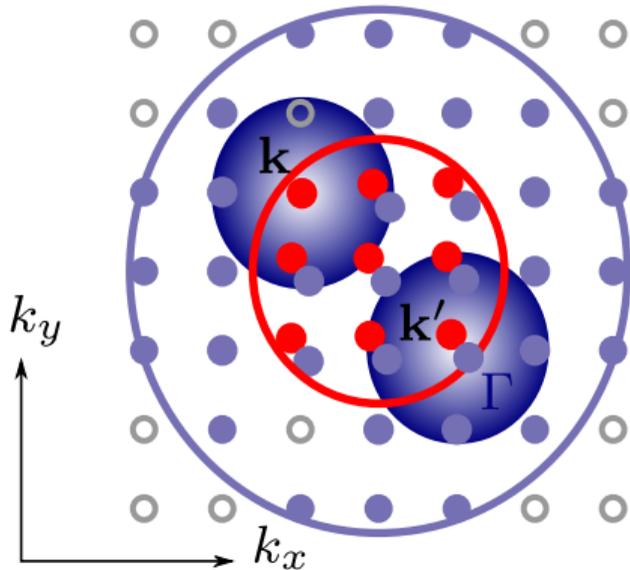
$$\int d^2\mathbf{k} n_{FD}(\mathbf{p}, \mathbf{r}) [1 - n_{FD}(\mathbf{p} + \hbar\mathbf{k}, \mathbf{r})]$$

Advantages

- ▶ valid for multi-level systems
- ▶ valid for generic geometries

Some Intuition

$$\frac{d\rho_{\mathbf{q}\mathbf{q}}^{ee}}{dt} = \sum_{\alpha} \sum_{\mathbf{k}} -2(1 - \rho_{\mathbf{k}\mathbf{k}}^{g_{\alpha}g_{\alpha}}) \rho_{\mathbf{q}\mathbf{q}}^{ee} \Gamma_{\alpha\alpha}^{\mathbf{k}\mathbf{q},\mathbf{q}\mathbf{k}}$$



Decay process

- ▶ $\Gamma_{\alpha\alpha}^{\mathbf{k}\mathbf{q},\mathbf{q}\mathbf{k}}$ mediates the decay of an atom e with momentum \mathbf{k}
- ▶ to g_{α} at momentum \mathbf{q}
- ▶ if $|\mathbf{k} - \mathbf{q}| \leq k_0$ (2D)

Consequences

- ▶ Highly imbalanced FS useful
- ▶ high occupation \rightarrow large Pauli blocking
- ▶ Dependence on k_F and T/T_F
- ▶ dimensionality matters!

Key Quantity of Interest

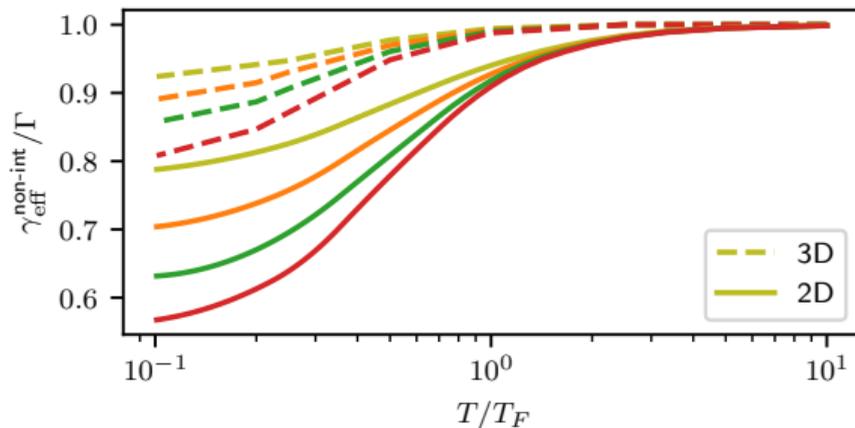
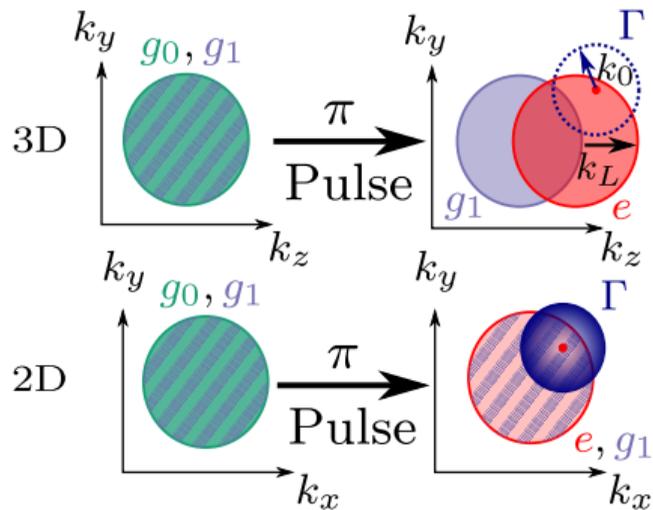
Effective Decay Rate

$$\gamma_{\text{eff}} = \lim_{t \rightarrow 0} \frac{\dot{N}_{ee}(t)}{N_{ee}(0)}$$

Advantages

- ▶ removes trivial N_{ee} scaling
- ▶ Our approximations are good at $t \rightarrow 0$
- ▶ simple, single parameter to study
- ▶ don't have to care about complex long-time dynamics

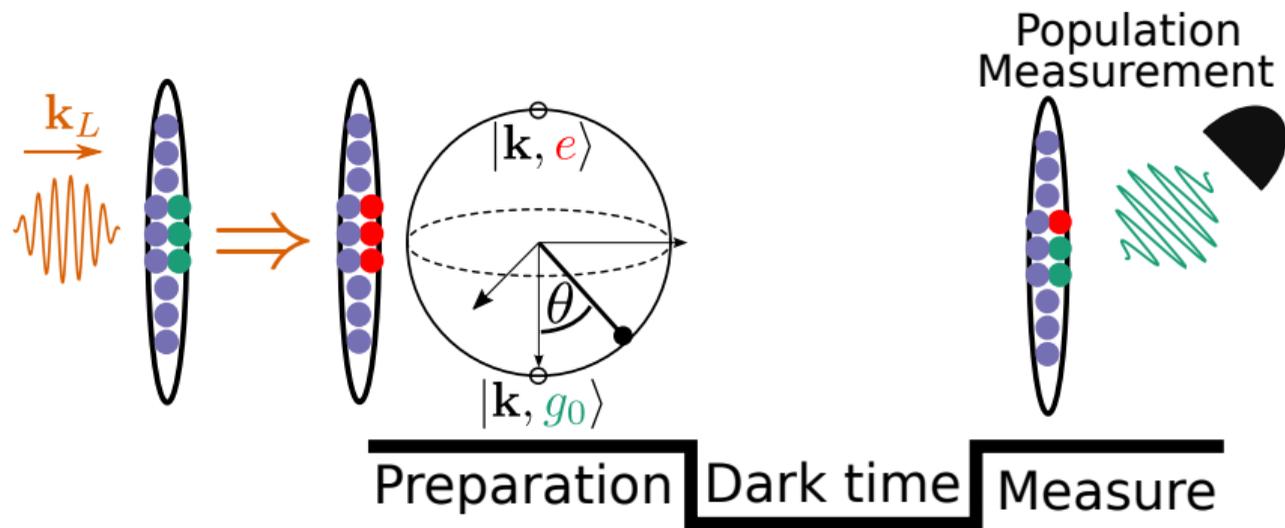
2D versus 3D



Why 2D is better

- ▶ no momentum kick in 2D
- ▶ mean-occupation higher in 2D due to density of states

Ramsey Protocol



Initial State:

$$\prod_{\mathbf{k}} n_0(\mathbf{k}) \left(\cos(\theta/2) |g_0, \mathbf{k}, n_{0,z}^{FD}\rangle + \sin(\theta/2) |e, \mathbf{k}, n_{0,z}\rangle \right) \otimes \prod_{\mathbf{k}} n_1^{FD}(\mathbf{k}) |g_1, \mathbf{k}, n_{0,z}\rangle$$

Interplay of Pauli blocking and interactions

EOM

$$\begin{aligned} \frac{d\rho_{\mathbf{q}\mathbf{q}}^{ee}}{dt} = & \sum_{\alpha} \sum_k -2(1 - \rho_{\mathbf{k}\mathbf{k}}^{g_{\alpha}g_{\alpha}}) \rho_{\mathbf{q}\mathbf{q}}^{ee} \Gamma_{\alpha\alpha}^{\mathbf{k}\mathbf{q},\mathbf{q}\mathbf{k}} \\ & + \sum_{\alpha,\beta} \sum_k i \left(\rho_{\mathbf{q}\mathbf{q}}^{eg_{\alpha}} \rho_{\mathbf{k}\mathbf{k}}^{g_{\beta}e} \mathcal{G}_{\alpha\beta}^{\mathbf{q}\mathbf{q},\mathbf{k}\mathbf{k}} - \rho_{\mathbf{q}\mathbf{q}}^{g_{\beta}e} \rho_{\mathbf{k}\mathbf{k}}^{eg_{\alpha}} \bar{\mathcal{G}}_{\alpha\beta}^{\mathbf{q}\mathbf{q},\mathbf{k}\mathbf{k}} \right) \end{aligned}$$

Interplay of Pauli blocking and interactions

EOM

$$\begin{aligned} \frac{d\rho_{\mathbf{q}\mathbf{q}}^{ee}}{dt} = & \sum_{\alpha} \sum_k -2(1 - \rho_{\mathbf{k}\mathbf{k}}^{g_{\alpha}g_{\alpha}}) \rho_{\mathbf{q}\mathbf{q}}^{ee} \Gamma_{\alpha\alpha}^{\mathbf{k}\mathbf{q},\mathbf{q}\mathbf{k}} \\ & + \sum_{\alpha,\beta} \sum_k i \left(\rho_{\mathbf{q}\mathbf{q}}^{eg_{\alpha}} \rho_{\mathbf{k}\mathbf{k}}^{g_{\beta}e} \mathcal{G}_{\alpha\beta}^{\mathbf{q}\mathbf{q},\mathbf{k}\mathbf{k}} - \rho_{\mathbf{q}\mathbf{q}}^{g_{\beta}e} \rho_{\mathbf{k}\mathbf{k}}^{eg_{\alpha}} \bar{\mathcal{G}}_{\alpha\beta}^{\mathbf{q}\mathbf{q},\mathbf{k}\mathbf{k}} \right) \end{aligned}$$

Initial State

$$\prod_{\mathbf{k}} n_0(\mathbf{k}) \left(\cos(\theta/2) |g_0, \mathbf{k}, n_{0,z}^{FD}\rangle + \sin(\theta/2) |e, \mathbf{k}, n_{0,z}\rangle \right) \otimes \prod_{\mathbf{k}} n_1^{FD}(\mathbf{k}) |g_1, \mathbf{k}, n_{0,z}\rangle$$

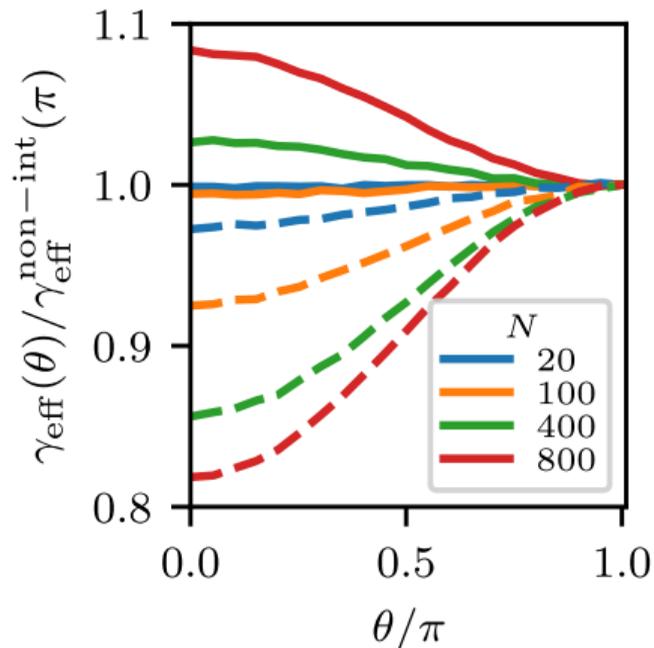
Pauli-Blocking

$$\rho^{g_0g_0} \sim \cos(\theta/2)^2$$

Cooperative Effects

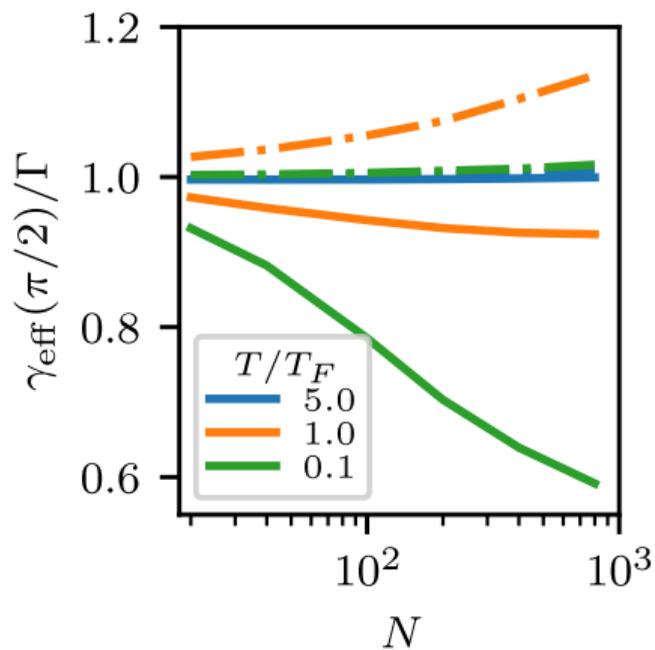
$$\rho^{ego} \rho^{g_0e} / \rho^{ee} \sim \cos(\theta/2)^2$$

Interplay of Pauli blocking and interactions



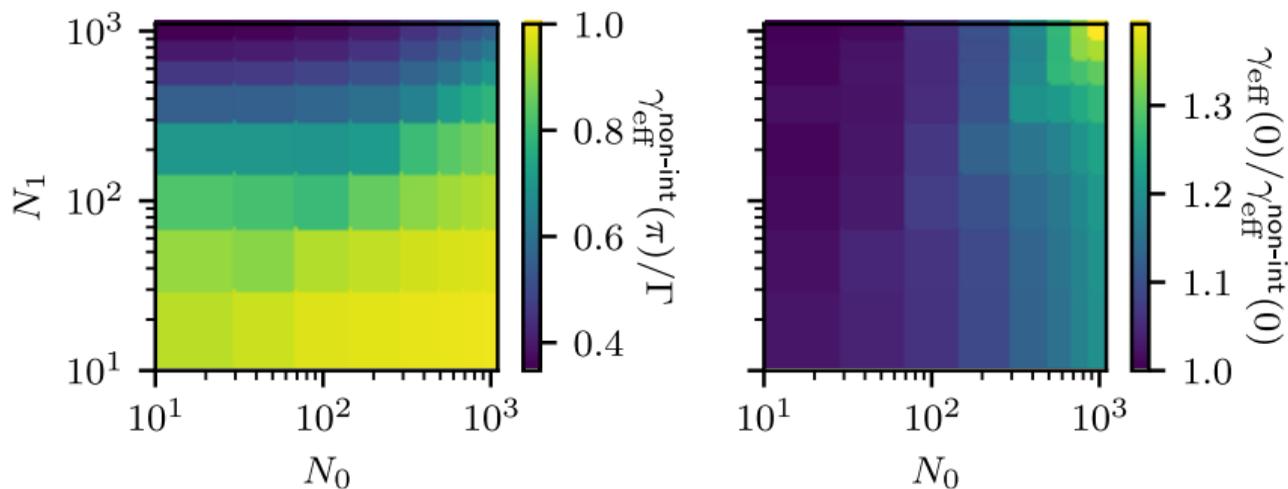
- ▶ Non-interacting ME shows stronger Pauli blocking at low θ
- ▶ interacting ME shows enhanced decay rate at low θ (for large enough N)

FA approximation atom versus ME in k-space



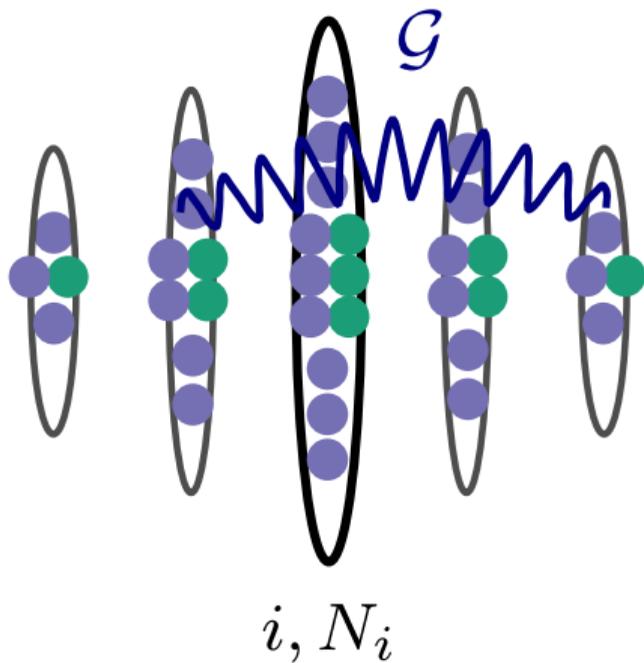
- ▶ Frozen atom captures super-radiant enhancement
- ▶ fails to capture Pauli blocking

Role of Imbalance



- ▶ Highly imbalanced FS allow strong Pauli blocking
- ▶ while minimizing cooperative effects

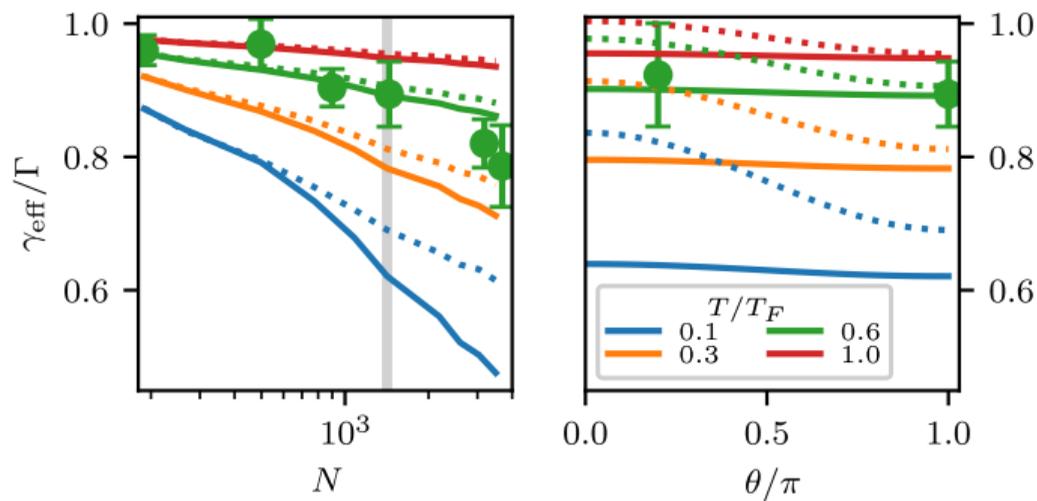
Extension to 1D lattice



1D optical lattice

- ▶ Slice 3D gas into stack of 2D pancakes
- ▶ Dipolar exchange between pancakes
- ▶ Distribution N_i over pancakes i
- ▶ $T/T_{F,i}$ different in each pancake
- ▶ in-plane size different in each pancake

Comparison with Experiment

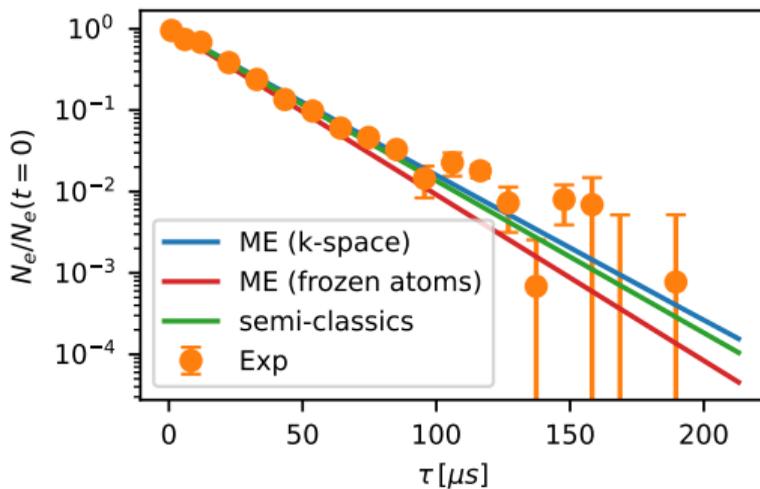
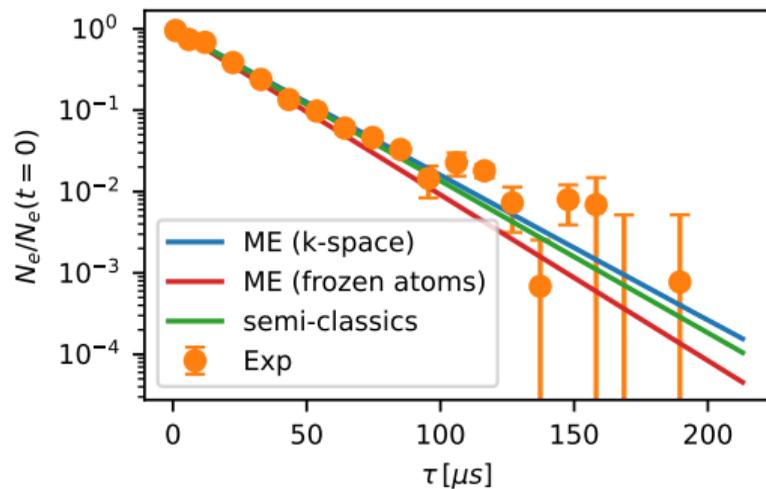


Exp Parameters

- ▶ $T/T_F = 0.6$
- ▶ $N_0 \sim 200$,
 $N_1 = 200, \dots, 3000$
- ▶ $\sim 7 - 10$ pancakes

- ▶ Theory consistent with experiment in highly imbalanced regime
- ▶ weak θ dependence for imbalanced case \rightarrow interactions not important

Real-Time Dynamics



- ▶ recall: $\Gamma^{-1} \sim 22\mu\text{s}$
- ▶ for $t\Gamma \lesssim 3 - 5$ consistent with single exponential
- ▶ $t \rightarrow 0$ limit well justified here

Conclusions

Summary

- ▶ Developed ME in k-space
- ▶ captures Pauli blocking and main cooperative effects
- ▶ applicable to multi-level and generic geometries
- ▶ consistent with experimental data

Open Questions

- ▶ Long time behaviour
- ▶ Future experiments to conclusively observe this!